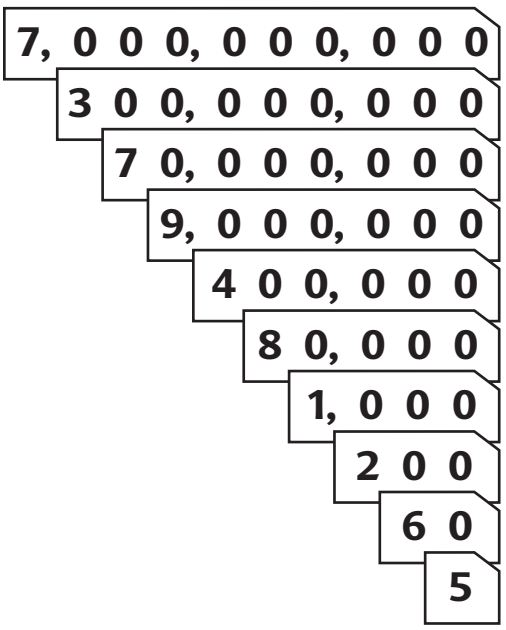
Unit 1 : Whole Numbers

Friendly Notes

Large Numbers

The number 7,379,481,265 can be represented as shown below.



Standard form: 7,379,481,265

Expanded form: 7,000,000,000 + 300,000,000 + 70,000,000 + 9,000,000 + 400,000 + 80,000 + 1,000 + 200 + 60 + 5

The number 7,379,481,265 written in words is seven billion, three hundred seventy-nine million, four hundred eighty-one thousand, two hundred sixty-five.

7,379,481,265
Starting from the right, each group of 3 digits forms a **period**. Commas separate the periods.

Billions			Millions			Thousands			Ones		
Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones
		7	3	7	9	4	8	1	2	6	5

The place value of 3 in 7,379,481,265 is hundred millions.

The digit 4 is in the hundred thousands place.

The value of the digit 9 is 9,000,000.

1 billion less than 7,379,481,265 is 6,379,481,265.

Which number is smaller, 8,425,678,900 or 8,455,678,837?

8,4**2**5,678,900

8,4**5**5,678,837

20 million is less than 50 million.

8,425,678,900 is smaller.

Approximation and Estimation

To round a number to a certain place value, we look at the digit in the next lower place value. If it is less than 5, we round down. If it is 5 or greater, we round up.

1. Round 1,345,826,917 to the nearest hundred million.

   What digit is in the hundred millions place?

   1,345,826,917

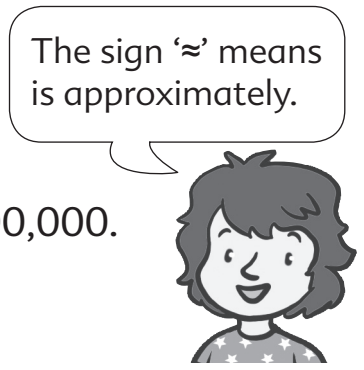
   What digit is in the next lower place value?

   1,345,826,917

   Do we round up or down?

   1,345,826,917 ≈ 1,300,000,000

   1,345,826,917 is approximately 1,300,000,000.



2. Round 426,839,105 to the nearest ten million.

   What digit is in the ten millions place?

   426,839,105

   What digit is in the next lower place value?

   426,839,105

   Do we round up or down?

   426,839,105 ≈ 430,000,000

   426,839,105 is approximately 430,000,000.

3. Round each number to the nearest ten million.
Then estimate the value of each of the following.

(a) 386,591,200 + 123,456,789

(b) 827,356,409 − 453,608,721

(a) 386,591,200 + 123,456,789
≈ 390,000,000 + 120,000,000
= 510,000,000

(b) 827,356,409 − 453,608,721
≈ 830,000,000 − 450,000,000
= 380,000,000

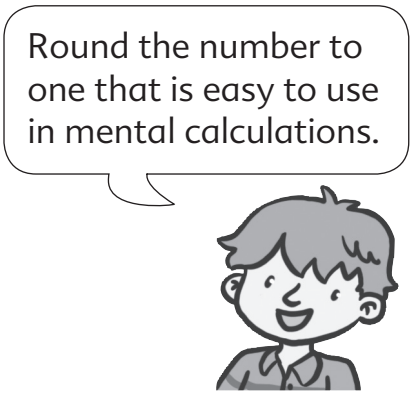
4. Estimate the value of each of the following.

(a) 11,021,040 × 5

(b) 84,976,314 ÷ 3

(a) 11,021,040 × 5
≈ 11,000,000 × 5
= 55,000,000

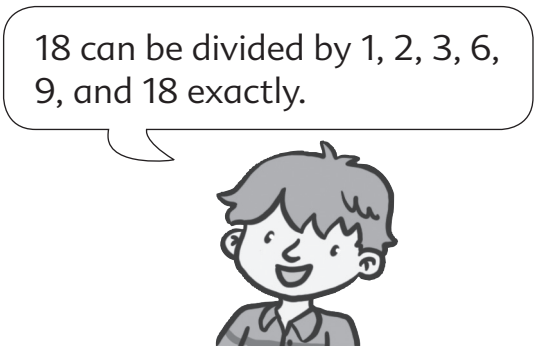
(b) 84,976,314 ÷ 3
≈ 90,000,000 ÷ 3
= 30,000,000



Factors and Multiples

Factors of a certain number divide the number exactly. A whole number can be expressed as a product of factors.

1. Find the factors of 18.



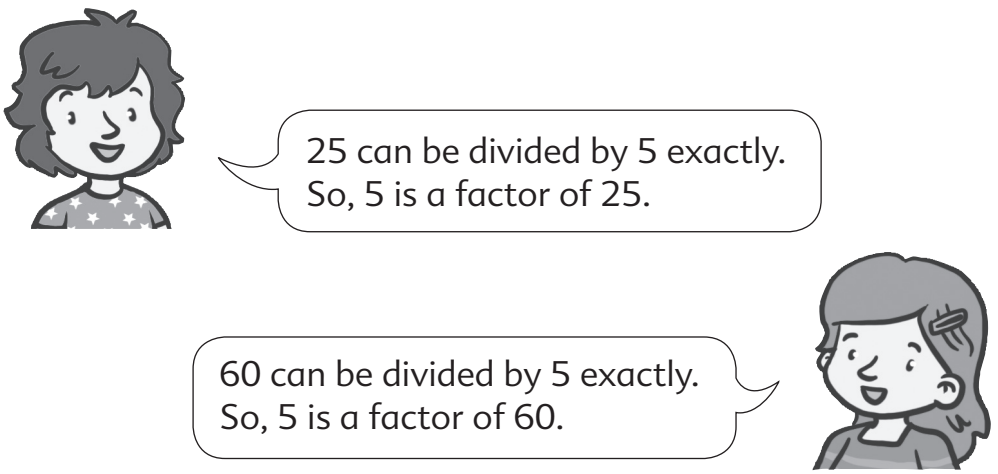
$18 = 1 \times 18$

$18 = 2 \times 9$

$18 = 3 \times 6$

The factors of 18 are 1, 2, 3, 6, 9, and 18.

2. Is 5 a common factor of 25 and 60?



As 5 is a factor of both 25 and 60, it is a **common factor** of 25 and 60.

Multiples of a number can be obtained by multiplying the number by whole numbers.

3. List the first four multiples of 6.

$1 \times 6 = 6$

$2 \times 6 = 12$

$3 \times 6 = 18$

$4 \times 6 = 24$

The first four multiples of 6 are 6, 12, 18, and 24.

4. Is 98 a common multiple of 2 and 7?

$$\begin{array}{r} 49 \\ 2\overline{)98} \\ 8 \\ \hline 18 \\ 18 \\ \hline 0 \end{array} \qquad \begin{array}{r} 14 \\ 7\overline{)98} \\ 7 \\ \hline 28 \\ 28 \\ \hline 0 \end{array}$$

Since 98 can be divided by both 2 and 7, 98 is a common multiple of 2 and 7.

Prime Factorization

A **prime number** is a whole number greater than 1 which has exactly two factors, 1 and the number itself.

A **composite number** is a whole number greater than 1 which has at least two factors that are not 1.

The numbers 0 and 1 are neither prime nor composite.

1. Identify
   (a) the prime numbers, and
   (b) the composite numbers, in the following.

   3, 4, 7, 21, 31, 47

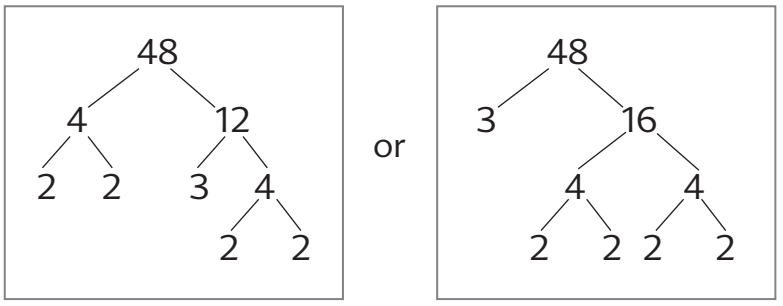
   (a) The prime numbers are 3, 7, 31, and 47.
   (b) The composite numbers are 4 and 21.

Prime factors are any factors of a number which are prime numbers.

Prime factorization is the process of factoring a composite number into its prime factors.

2. Express 48 as a product of prime factors only.

   Method 1: Use a factor tree.



$48 = 2 \times 2 \times 2 \times 2 \times 3$
$= 2^4 \times 3$

Method 2: Use continuous division, starting with the lowest prime number that is a factor.

2	48
2	24
2	12
2	6
3	3
	1

$48 = 2 \times 2 \times 2 \times 2 \times 3$
$= 2^4 \times 3$

In 2^4, 2 is the base and 4 is the exponent.
The **exponent** tells us how many times the base is used as a factor.

3. Write each of the following using exponents.

(a) $5 \times 5 \times 5 \times 5 \times 5 \times 5$

(b) $3 \times 3 \times 3 \times 3 \times 7 \times 7 \times 7$

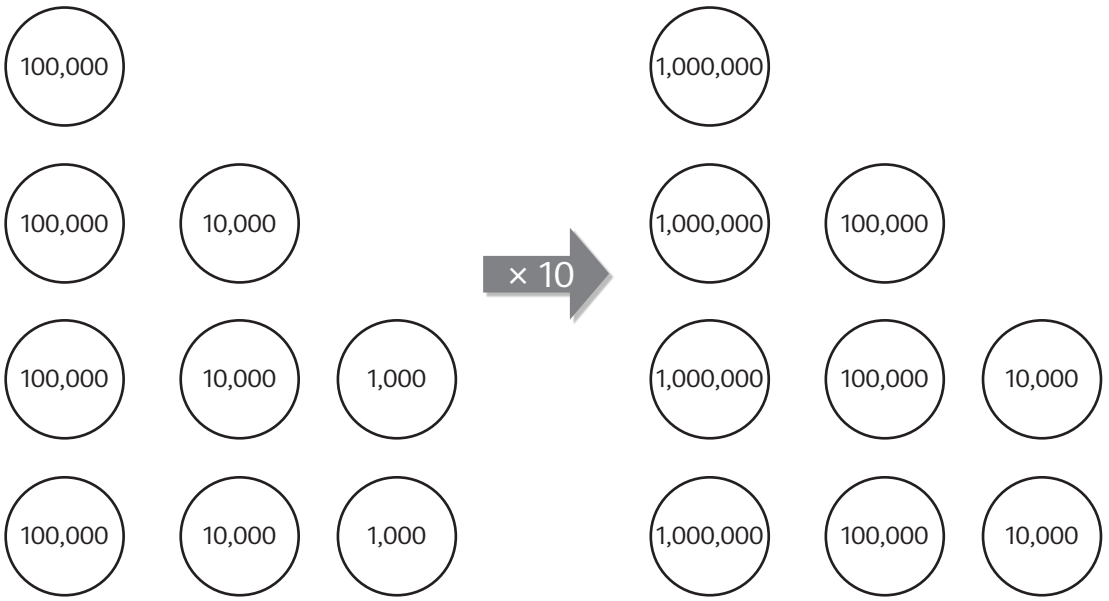
(a) $5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^6$

(b) $3 \times 3 \times 3 \times 3 \times 7 \times 7 \times 7 = 3^4 \times 7^3$

Multiplying by Tens, Hundreds, or Thousands

When multiplying a whole number by 10, the number is increased 10 times.

1. Multiply 432,000 by 10.



$432{,}000 \times \mathbf{10} = 4{,}320{,}00\mathbf{0}$

$432{,}000 \times 10^{\mathbf{1}} = 4{,}320{,}00\mathbf{0}$

2. Multiply 432,000 by 20.

$$\begin{aligned} 432{,}000 \times 2\mathbf{0} &= 432{,}000 \times 2 \times 1\mathbf{0} \\ &= 864{,}000 \times 1\mathbf{0} \\ &= 8{,}640{,}00\mathbf{0} \end{aligned}$$

Multiply 432,000 by 2 first.
$432{,}000 \times 2 = 864{,}000$

When multiplying a whole number by 100, the number is increased 100 times.

3. Multiply 150,000 by 100.

$150{,}000 \times 1\mathbf{00} = 15{,}000{,}0\mathbf{00}$

$150{,}000 \times 10^2 = 15{,}000{,}0\mathbf{00}$

4. Multiply 150,000 by 300.

$$\begin{aligned} 150{,}000 \times 3\mathbf{00} &= 150{,}000 \times 3 \times 1\mathbf{00} \\ &= 450{,}000 \times 1\mathbf{00} \\ &= 45{,}000{,}0\mathbf{00} \end{aligned}$$

When multiplying a whole number by 1,000, the number is increased 1,000 times.

5. Multiply 240,000 by 1,000.

$240{,}000 \times 1{,}\mathbf{000} = 240{,}000{,}\mathbf{000}$

$240{,}000 \times 10^3 = 240{,}000{,}\mathbf{000}$

6. Multiply 240,000 by 4,000.

$$\begin{aligned} 240{,}000 \times 4{,}\mathbf{000} &= 240{,}000 \times 4 \times 1{,}\mathbf{000} \\ &= 960{,}000 \times 1{,}\mathbf{000} \\ &= 960{,}000{,}\mathbf{000} \end{aligned}$$

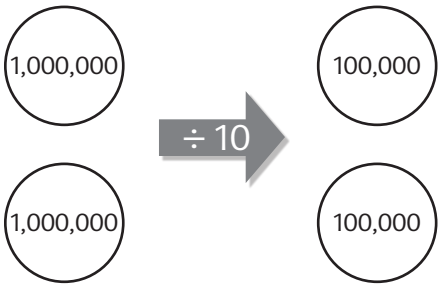
7. Estimate the value of $108{,}000 \times 2{,}015$.

$$\begin{aligned} 108{,}000 \times 2{,}015 &\approx 110{,}000 \times 2{,}\mathbf{000} \\ &= 220{,}000{,}\mathbf{000} \end{aligned}$$

Dividing by Tens, Hundreds, or Thousands

When dividing a whole number by 10, the number is reduced 10 times.

1. Divide 2,000,000 by 10.

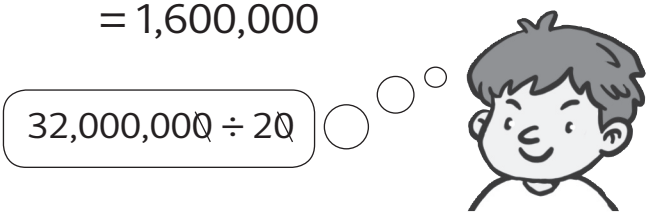


$2{,}000{,}00\mathbf{0} \div 1\mathbf{0} = 200{,}000$

$2{,}000{,}00\mathbf{0} \div 10^{1} = 200{,}000$

2. Divide 32,000,000 by 20.

$32{,}000{,}00\mathbf{0} \div 2\mathbf{0} = 32{,}000{,}00\mathbf{0} \div 1\mathbf{0} \div 2$
$= 3{,}200{,}000 \div 2$
$= 1{,}600{,}000$



When dividing a whole number by 100, the number is reduced 100 times.

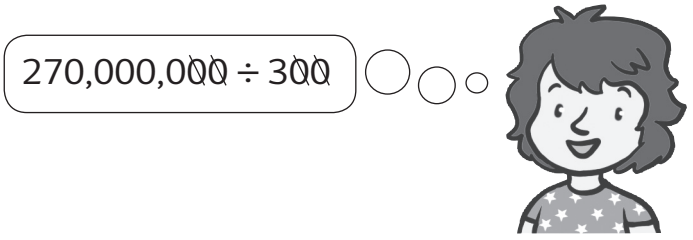
3. Divide 600,000,000 by 100.

$600{,}000{,}0\mathbf{00} \div 1\mathbf{00} = 6{,}000{,}000$

$600{,}000{,}0\mathbf{00} \div 10^{2} = 6{,}000{,}000$

4. Divide 270,000,000 by 300.

$270{,}000{,}0\mathbf{00} \div 3\mathbf{00} = 270{,}000{,}0\mathbf{00} \div 1\mathbf{00} \div 3$
$= 2{,}700{,}000 \div 3$
$= 900{,}000$



When dividing a whole number by 1,000, the number is reduced 1,000 times.

5. Divide 800,000,000 by 1,000.

$800{,}000{,}\mathbf{000} \div 1{,}\mathbf{000} = 800{,}000$

$800{,}000{,}\mathbf{000} \div 10^3 = 800{,}000$

6. Divide 750,000,000 by 5,000.

$750{,}000{,}\mathbf{000} \div 5{,}\mathbf{000} = 750{,}000{,}\mathbf{000} \div 1{,}\mathbf{000} \div 5$
$= 750{,}000 \div 5$
$= 150{,}000$

7. Estimate the value of 5,318,300,200 ÷ 6,955.

$5{,}318{,}300{,}200 \div 6955 \approx 5{,}600{,}000{,}\mathbf{000} \div 7{,}\mathbf{000}$
$= 800{,}000$

Unit 2 : More Calculations with Whole Numbers

Friendly Notes

Order of Operations

Do multiplication or division from left to right, then addition or subtraction from left to right.

Compute the expression in parentheses first, if any.

1. Find the value of $6 \times 8 + 30 \div 5 - 4 \times 9$.

$$\mathbf{6 \times 8} + \mathbf{30 \div 5} - \mathbf{4 \times 9} = 48 + 6 - 36$$
$$= 18$$

If the expression in the parentheses has different kinds of operations, use the order of operations.

2. Find the value of $600 \div (5 + 11 \times 5) - 8$.

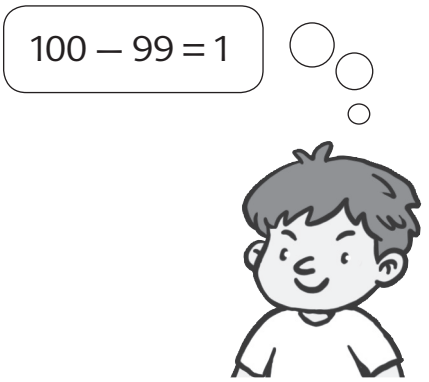
$$600 \div (5 + \mathbf{11 \times 5}) - 8 = 600 \div (\mathbf{5 + 55}) - 8$$
$$= \mathbf{600 \div 60} - 8$$
$$= 10 - 8$$
$$= 2$$

Methods for Mental Calculation

When we add numbers close to 100 to another number, we can add 100 first and then subtract the difference between the number and 100.

1. Add 455 and 99.

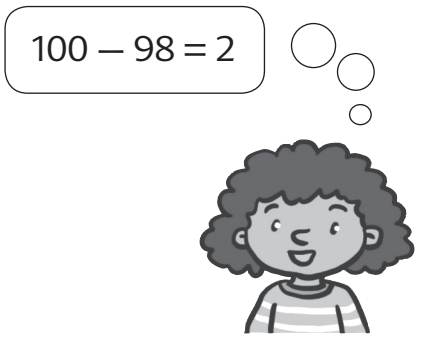
$455 + 99 = 455 + 100 - 1$
$= 555 - 1$
$= 554$



When we subtract numbers close to 100 from another number, we can subtract 100 first and then add the difference between the number and 100.

2. Subtract 98 and 367.

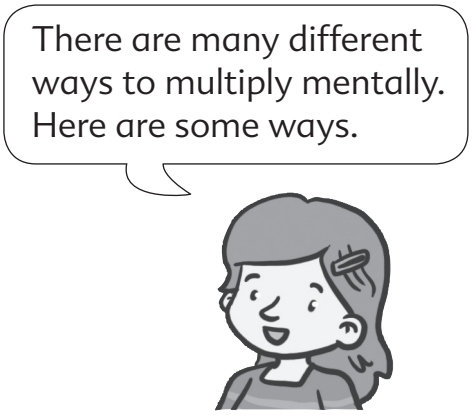
$367 - 98 = 367 - 100 + 2$
$= 267 + 2$
$= 269$



When we multiply mentally, we can multiply the tens, multiply the ones, and then add the products.

3. Multiply 85 by 6.

$$\begin{aligned} 85 \times 6 &= 80 \times 6 + 5 \times 6 \\ &= 480 + 30 \\ &= 510 \end{aligned}$$



4. Multiply 85 by 60.

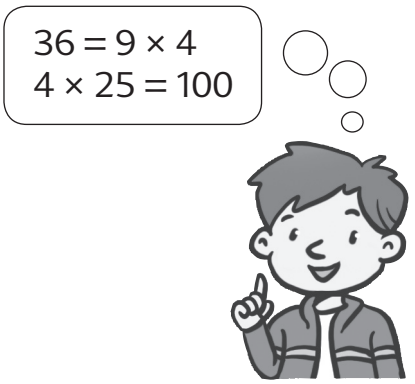
$$\begin{aligned} 85 \times 60 &= 85 \times 6 \times 10 \\ &= 510 \times 10 \\ &= 5{,}100 \end{aligned}$$

5. Multiply 67 by 99.

$$\begin{aligned} 67 \times 99 &= 67 \times 100 - 67 \\ &= 6{,}700 - 67 \\ &= 6{,}633 \end{aligned}$$

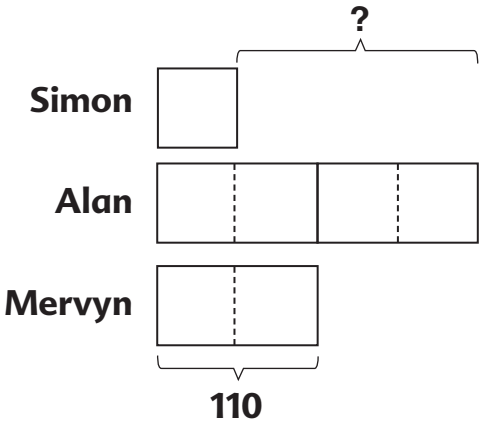
6. Multiply 36 by 25.

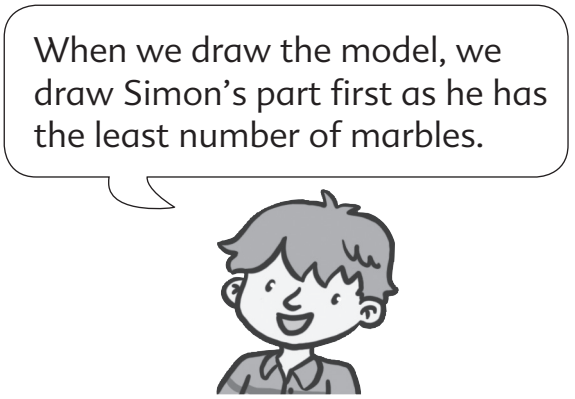
$$\begin{aligned} 36 \times 25 &= 9 \times 4 \times 25 \\ &= 9 \times 100 \\ &= 900 \end{aligned}$$



Word Problems

1. Mervyn has 110 marbles. Alan has 2 times as many marbles as Mervyn and 4 times as many marbles as Simon. How many more marbles does Alan have than Simon?





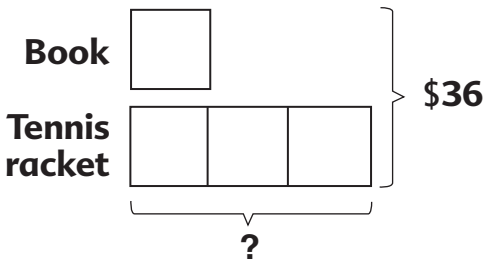
2 units = 110 marbles

1 unit = 55 marbles

3 units = 165 marbles

Alan has 165 more marbles than Simon.

2. A tennis racket costs 3 times as much as a book. If the total cost of the book and the tennis racket is $36, find the cost of the tennis racket.



4 units = $36

1 unit = $9

3 units = $27

The cost of the tennis racket is $27.

Multiplication by a 2-Digit Whole Number

1. Multiply 57 by 60.

Method 1:

$$\begin{aligned} 57 \times 60 &= 57 \times 6 \times 10 \\ &= 342 \times 10 \\ &= 3{,}420 \end{aligned}$$

Method 2:

$$\begin{array}{r} {\scriptstyle 4}\,5\,7 \\ \times \quad 6\,0 \\ \hline 3{,}420 \end{array}$$

Method 3:

	50	7
60	$50 \times 60 = 3{,}000$	$7 \times 60 = 420$

$$\begin{array}{r} 3{,}000 \\ + \quad 420 \\ \hline 3{,}420 \end{array}$$

2. Multiply 368 by 25.

$$\begin{array}{r} 3\,6\,8 \\ \times \quad 2\,5 \\ \hline 1{,}8\,4\,0 \leftarrow 368 \times 5 \\ 7{,}3\,6\,0 \leftarrow 368 \times 20 \\ \hline 9{,}2\,0\,0 \end{array}$$

	300	60	8
20	$300 \times 20 = 6{,}000$	$60 \times 20 = 1{,}200$	$8 \times 20 = 160$
5	$300 \times 5 = 1{,}500$	$60 \times 5 = 300$	$8 \times 5 = 40$

$$\begin{array}{r} 6{,}000 \\ 1{,}200 \\ 160 \\ 1{,}500 \\ 300 \\ + \quad 40 \\ \hline 9{,}200 \end{array}$$

Division by a 2-Digit Whole Number

1. Divide 98 by 36.

$$36\overline{)\begin{array}{r}2\\98\\72\\\hline26\end{array}}$$

$$40\overline{)\begin{array}{r}2\\98\end{array}}$$

$40 \times 2 = 80$ $40 \times 3 = 120$

The estimated quotient is 2.

$30 \times ? = 90$

	30	6
2	60	12
	72	

$$\begin{array}{rr} & 98\\ - & 72\\ \hline & 26 < 36\end{array}$$

$98 \div 36 = 2$ R 26

$36 \times 2 + 36 = 98$

2. Divide 225 by 25.

$$30\overline{)\begin{array}{r}7\\225\end{array}}$$

The estimated quotient is 7.

$$25\overline{)\begin{array}{r}7\\225\\175\\\hline50\end{array}} \rightarrow 25\overline{)\begin{array}{r}9\\225\\225\\\hline0\end{array}}$$

The estimated quotient is too small. Try 9.

$20 \times ? = 200$

	20	5
7	140	35
+ 2	40	10
9	225	

$$\begin{array}{rr} & 225\\ - & 175\\ \hline & 50\\ - & 50\\ \hline & 0\end{array}$$

$225 \div 25 = 9$

$25 \times 9 = 225$

Unit 3 : Fractions

Friendly Notes

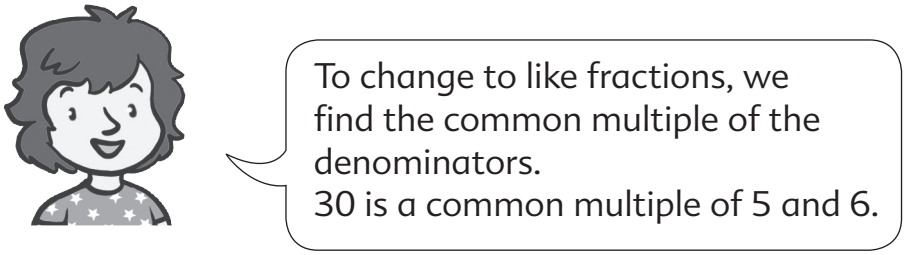
Looking Back

To compare fractions, we change them to **like fractions**. Like fractions are fractions with a common denominator. For like fractions, the greater the numerator, the greater the fraction.

Which is greater, $\frac{4}{5}$ or $\frac{5}{6}$?

$\frac{4}{5} \overset{\times 6}{=} \frac{24}{30}$ ($\times 6$)

$\frac{5}{6} \overset{\times 5}{=} \frac{25}{30}$ ($\times 5$)



$\frac{25}{30}$ is greater than $\frac{24}{30}$.

So, $\frac{5}{6}$ is greater than $\frac{4}{5}$.

Fractions and Division

Find the value of $34 \div 8$.

Method 1:

$$34 \div 8 = 4\frac{2}{8}$$
$$= 4\frac{1}{4}$$

$$\begin{array}{r} 4 \\ 8\overline{)\,3\,4} \\ 3\,2 \\ \hline 2 \end{array}$$

Method 2:

$$34 \div 8 = \frac{34}{8}$$
$$= \frac{17}{4}$$
$$= \frac{16}{4} + \frac{1}{4}$$
$$= 4\frac{1}{4}$$

Reduce $\frac{34}{8}$ to its simplest form.

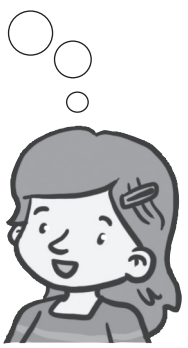
Addition and Subtraction of Unlike Fractions

Unlike fractions are fractions which do not have the same denominator. When adding or subtracting unlike fractions, we change them to like fractions first.

1. Add $\frac{2}{3}$ and $\frac{3}{10}$.

$$\frac{2}{3} + \frac{3}{10} = \frac{20}{30} + \frac{9}{30}$$
$$= \frac{29}{30}$$

$\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \ldots, \frac{20}{30}$
$\frac{3}{10}, \frac{6}{20}, \frac{9}{30}, \ldots$
30 is a common multiple of 3 and 10.



2. Subtract $\frac{2}{3}$ from $\frac{4}{5}$.

$$\frac{4}{5} - \frac{2}{3} = \frac{12}{15} - \frac{10}{15}$$
$$= \frac{2}{15}$$

$\frac{4}{5}, \frac{8}{10}, \frac{12}{15}, \ldots$
$\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \frac{10}{15}, \ldots$
15 is a common multiple of 5 and 3.

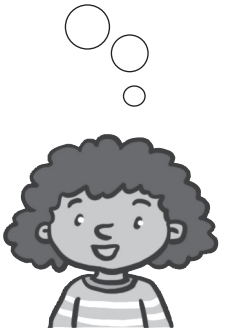
Addition and Subtraction of Mixed Numbers

When adding or subtracting mixed numbers, we add or subtract the whole numbers first and then the fractions.

1. Add $3\frac{1}{4}$ and $2\frac{3}{8}$.

$$3\frac{1}{4} + 2\frac{3}{8} = 5\frac{1}{4} + \frac{3}{8}$$
$$= 5\frac{2}{8} + \frac{3}{8}$$
$$= 5\frac{5}{8}$$

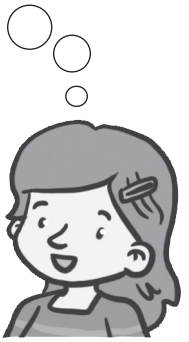
$3\frac{1}{4} \xrightarrow{+2} 5\frac{1}{4} \xrightarrow{+\frac{3}{8}} 5\frac{5}{8}$



2. Subtract $1\frac{5}{6}$ from $4\frac{3}{4}$.

$$4\frac{3}{4} - 1\frac{5}{6} = 3\frac{3}{4} - \frac{5}{6}$$
$$= 3\frac{9}{12} - \frac{10}{12}$$
$$= 2\frac{21}{12} - \frac{10}{12}$$
$$= 2\frac{11}{12}$$

$$3\frac{9}{12} = 2 + 1 + \frac{9}{12}$$
$$= 2 + \frac{12}{12} + \frac{9}{12}$$
$$= 2\frac{21}{12}$$



$$4\frac{3}{4} \xrightarrow{-1} 3\frac{3}{4} \xrightarrow{-\frac{5}{6}} 2\frac{11}{12}$$

Product of a Fraction and a Whole Number

When multiplying a fraction by a whole number, we multiply the whole number by the numerator of the fraction.

1. Multiply 9 by $\frac{3}{10}$.

$$9 \times \frac{3}{10} = \frac{27}{10}$$
$$= 2\frac{7}{10}$$

$$9 \times \frac{3}{10} = \frac{3 \times 9}{10}$$
$$9 \times \frac{3}{10} = \frac{9 \times 3}{10}$$

2. Find $\frac{3}{5}$ of a liter in milliliters.

$\frac{3}{5}$ of a liter $= \frac{3}{5} \times 1$ L

$= \frac{3}{5} \times 1{,}000$ ml

$= \frac{3 \times 1{,}000}{5}$

$= \frac{3{,}000}{5}$

$= 600$ ml

1 L = 1,000 ml

3. Find $\frac{3}{4}$ of a meter in centimeters.

$\frac{3}{4}$ of a meter $= \frac{3}{4} \times 1$ m

$= \frac{3}{4} \times 100$ cm

$= \frac{3 \times 100}{4}$

$= \frac{300}{4}$

$= 75$ cm

1 m = 100 cm

Word Problems

There are 42 children in a class. $\frac{1}{6}$ of them wear glasses.

How many children do not wear glasses?

Method 1:

$1 - \frac{1}{6} = \frac{5}{6}$

First, I find what fraction of the children do not wear glasses.

$\frac{5}{6}$ of the children do not wear glasses.

$\frac{5}{6} \times 42 = \frac{5 \times \cancel{42}^{7}}{1\cancel{6}}$

$= 35$

35 children do not wear glasses.

Method 2:

Number of children who wear glasses $= \frac{1 \times \cancel{42}^{7}}{1\cancel{6}}$

$= 7$

Number of children who do not wear glasses $= 42 - 7$

$= 35$

Method 3:

6 units = 42 children

1 unit = 7 children

Number of children who do not wear glasses = 5 units

$= 5 \times 7$

$= 35$

Unit 4 : Multiply and Divide Fractions

Friendly Notes

Product of Fractions

When multiplying a fraction by another fraction, we multiply the numerators and the denominators separately.

1. Multiply $\frac{2}{3}$ by $\frac{7}{10}$.

Method 1:

$$\frac{2}{3} \times \frac{7}{10} = \frac{{}^{1}\cancel{2} \times 7}{3 \times \cancel{10}_{5}}$$
$$= \frac{7}{15}$$

Method 2:

$$\frac{{}^{1}\cancel{2}}{\cancel{3}} \times \frac{7}{\cancel{10}_{5}} = \frac{7}{15}$$

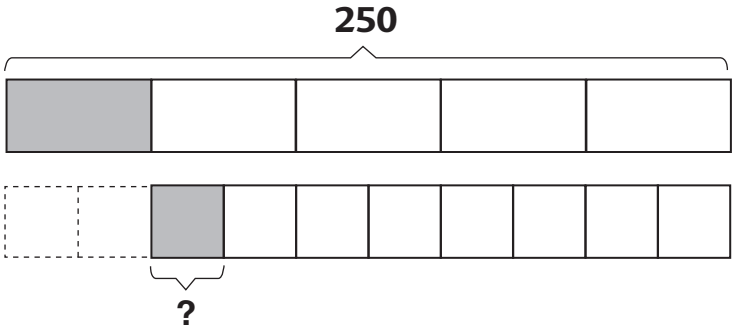
2. $\frac{3}{4}$ of a class are girls. $\frac{1}{6}$ of the girls have long hair. What fraction of the class are girls with long hair?

$$\frac{1}{6} \times \frac{3}{4} = \frac{3}{24}$$
$$= \frac{1}{8}$$

$\frac{1}{8}$ of the class are girls with long hair.

Word Problems

A book has 250 pages. Melissa read $\frac{1}{5}$ of the book on Friday and $\frac{1}{8}$ of the remaining pages on Saturday. How many pages did she read on Saturday?



Method 1:

$1 - \frac{1}{5} = \frac{4}{5}$

She had $\frac{4}{5}$ of the pages in the book left to read on Friday.

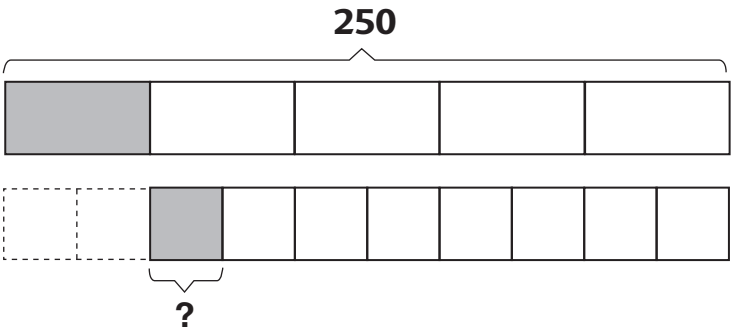
$\frac{1}{\cancel{8}_{2}} \times \frac{\cancel{4}^{1}}{5} = \frac{1}{10}$

She read $\frac{1}{10}$ of the pages in the book on Saturday.

$\frac{1}{10} \times 250 = 25$

Melissa read 25 pages on Saturday.

Method 2:



$1 - \frac{1}{5} = \frac{4}{5}$

She had $\frac{4}{5}$ of the pages in the book left to read on Friday.

$\frac{4}{\cancel{5}_{1}} \times \cancel{250}^{50} = 200$

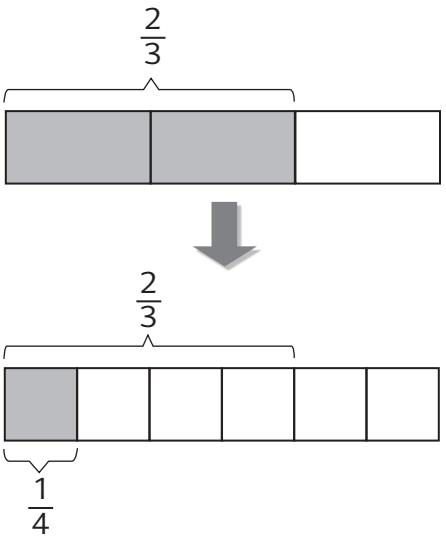
She had 200 pages left on Friday.

$\frac{1}{\cancel{8}_{1}} \times \cancel{200}^{25} = 25$

Melissa read 25 pages on Saturday.

Dividing a Fraction by a Whole Number

1. Divide $\frac{2}{3}$ by 4.



Divide $\frac{2}{3}$ into 4 equal parts.
Each part is $\frac{1}{4}$ of $\frac{2}{3}$.

$\frac{2}{3} \div 4 = \frac{2}{3} \times \frac{1}{4}$

$= \frac{2}{12}$

$= \frac{1}{6}$

2. 3 children shared $\frac{3}{5}$ of a pie equally. How much pie did each of them get?

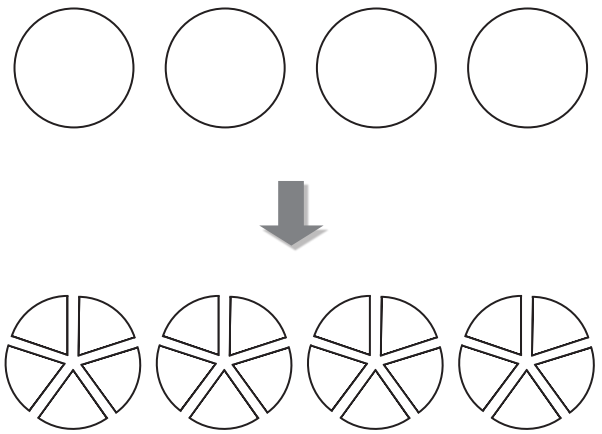
$$\frac{3}{5} \div 3 = \frac{3}{5} \times \frac{1}{3}$$
$$= \frac{3}{15}$$
$$= \frac{1}{5}$$

Each of them got $\frac{1}{5}$ of the pie.

Dividing by 3 is the same as multiplying by $\frac{1}{3}$.

Dividing by a Fraction

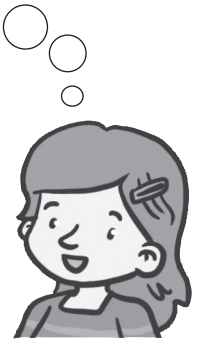
1. Rita bought 4 pies. She cut each pie into fifths. How many pieces of pie did she have?



There are 5 fifths in 1 whole. There are 20 fifths in 4 wholes.

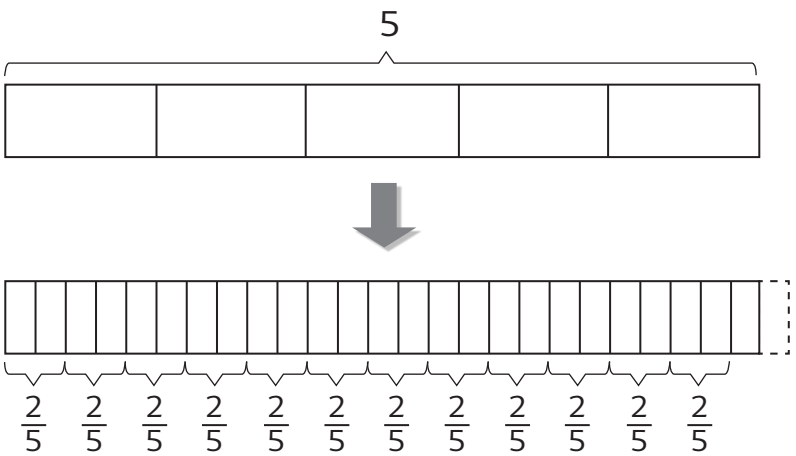
$4 \div \frac{1}{5} = 4 \times 5$

$= 20$



Rita had 20 pieces of pie.

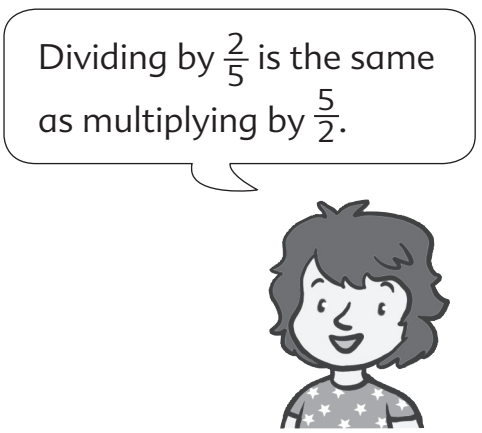
2. Divide 5 by $\frac{2}{5}$.



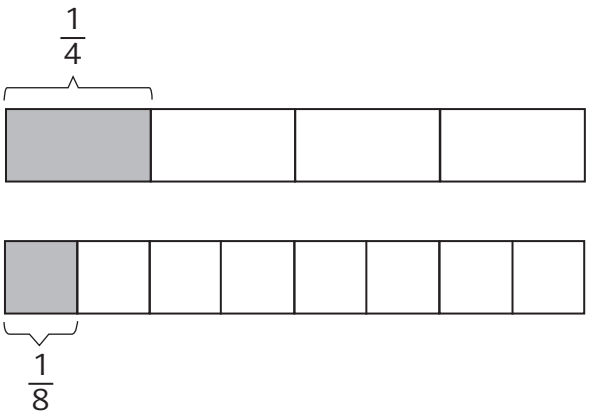
$5 \div \frac{2}{5} = 5 \times \frac{5}{2}$

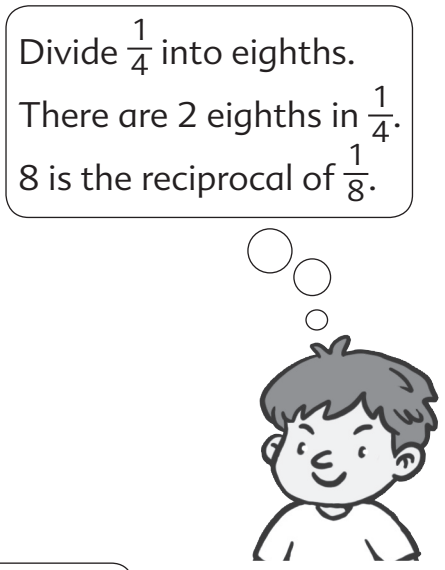
$= \frac{25}{2}$

$= 12\frac{1}{2}$

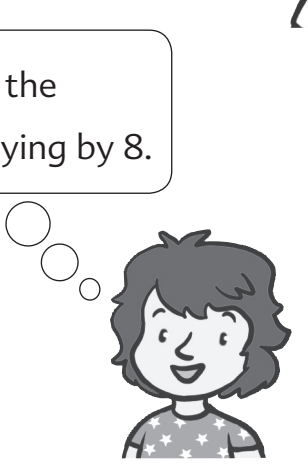


3. Divide $\frac{1}{4}$ by $\frac{1}{8}$.





$$\frac{1}{4} \div \frac{1}{8} = \frac{1}{4} \times 8$$
$$= \frac{8}{4}$$
$$= 2$$



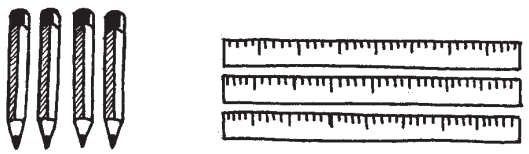
Unit 6 : Ratio

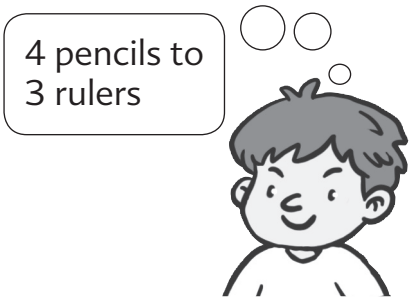
Friendly Notes

Finding Ratio

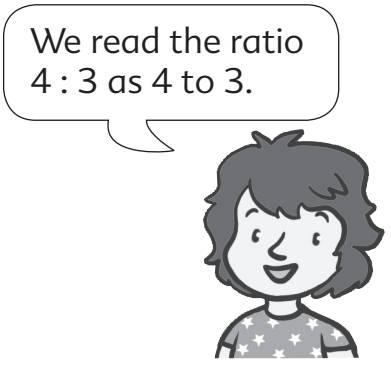
Ratio is a comparison of two or more similar quantities.

There are 4 pencils and 3 rulers.





The ratio of the number of pencils to the number of rulers is 4 : 3.



Equivalent Ratios

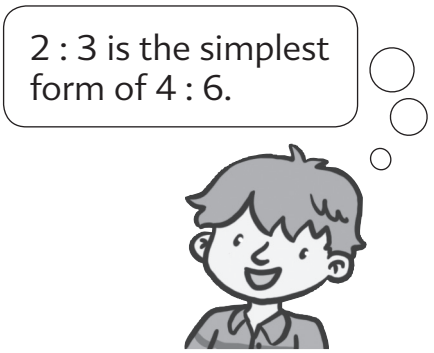
Ratio has no units.

When we divide or multiply both quantities in a ratio by the same number, we obtain equivalent ratios.

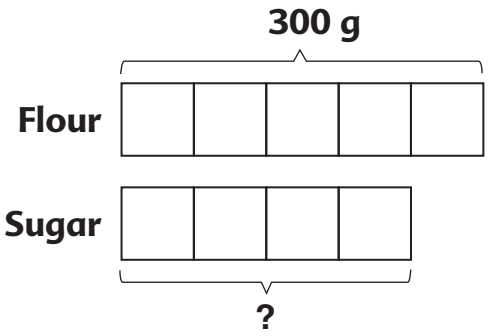
1. Find an equivalent ratio for 4 : 6.

$$\begin{array}{c} 4 : 6 \\ \div 2 \downarrow \quad \downarrow \div 2 \\ 2 : 3 \end{array}$$

2 : 3 is an equivalent ratio for 4 : 6.



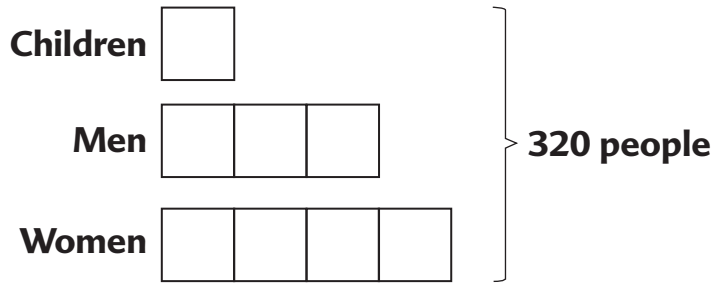
2. The ratio of flour to sugar used to make a pie is 5 : 4. If the mass of flour used is 300 g, find the mass of sugar used in making the pie.



5 units = 300 g
1 unit = 60 g
4 units = 240 g
The mass of sugar used is 240 g.

Combining Three Quantities

The number of children to the number of men to the number of women at a concert is 1 : 3 : 4. If there were 320 people altogether at the concert, find the number of men at the concert.



8 units = 320
1 unit = 40
3 units = 120
There were 120 men at the concert.

Unit 7 : Decimals

Friendly Notes

Tenths, Hundredths, and Thousandths

1 one = 10 tenths
1 tenth = 10 hundredths
1 hundredth = 10 thousandths

1. Write 42 tenths as a decimal.

   42 tenths = 40 tenths + 2 tenths
   = 4 ones + 2 tenths
   = 4 + 0.2
   = 4.2

2. Find the value of the digit 6 in 2.563.

   2.563 = 2 ones 5 tenths 6 hundredths 3 thousandths
   = 2 + 0.5 + 0.06 + 0.003

   The digit 6 is in the hundredths place.

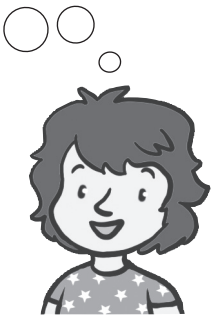
   The value of the digit 6 is 0.06.

2.563 has 3 decimal places. The tenths place, hundredths place, and thousandths place are called decimal places.

3. What number is 0.001 more than 5.083?

5.083 = 5 ones + 8 hundredths + 3 thousandths

0.001 = 1 thousandth
3 thousandths + 1 thousandth = 4 thousandths



5.084 is 0.001 more than 5.083.

4. Which is smaller, 8.246 or 8.232?

Ones	Tenths	Hundredths	Thousandths
8	2	**4**	6
8	2	**3**	2

3 hundredths is smaller than 4 hundredths.
So, 8.232 is smaller.

5. Which is greater, 51.378 or 51.379?

Tens	Ones	Tenths	Hundredths	Thousandths
5	1	3	7	**8**
5	1	3	7	**9**

9 thousandths is greater than 8 thousandths.
So, 51.379 is greater.

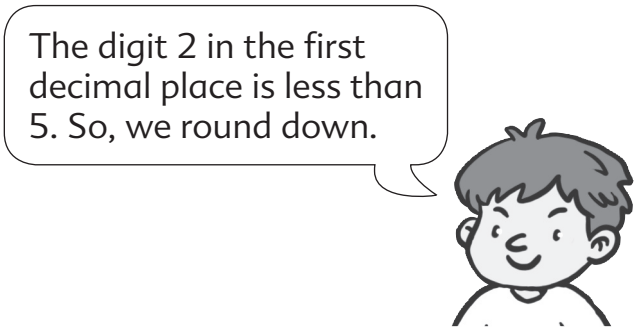
Approximation

To round a decimal to the nearest whole number, we look at the digit in the first decimal place. If it is 5 or greater, we round up. If it is less than 5, we round down.

1. Round 4.2 m to the nearest meter.

4.2

4.2 m ≈ 4 m

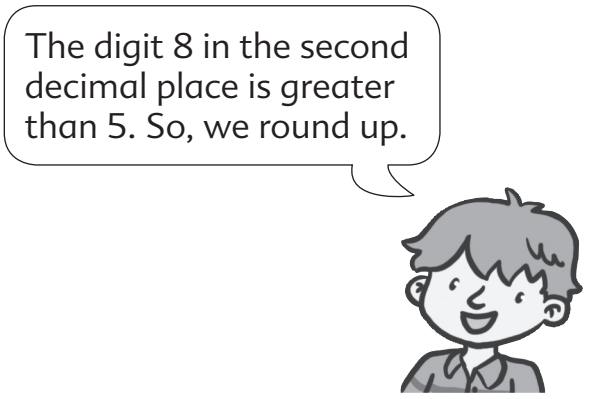


To round a decimal to 1 decimal place, we look at the digit in the second decimal place. If it is 5 or greater, we round up. If it is less than 5, we round down.

2. Round 6.28 to 1 decimal place.

6.28

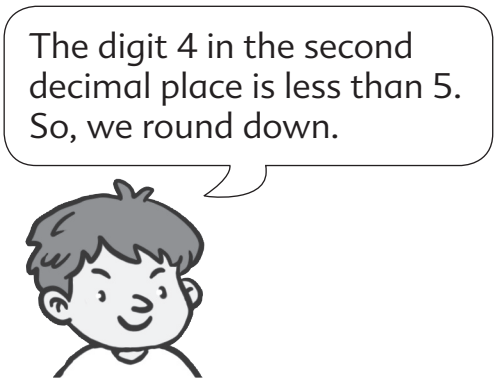
6.28 ≈ 6.3



3. Round 10.845 to 1 decimal place.

10.845

10.845 ≈ 10.8



To round a decimal to 2 decimal places, we look at the digit in the third decimal place. If it is 5 or greater, we round up. If it is less than 5, we round down.

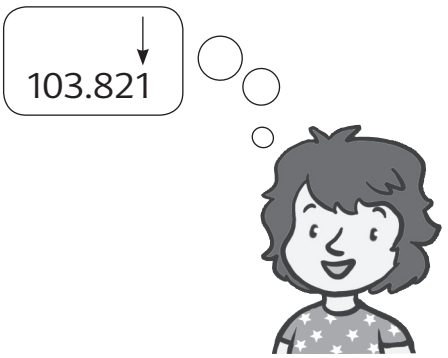
4. Round 15.649 to 2 decimal places.

15.649

15.649 ≈ 15.65

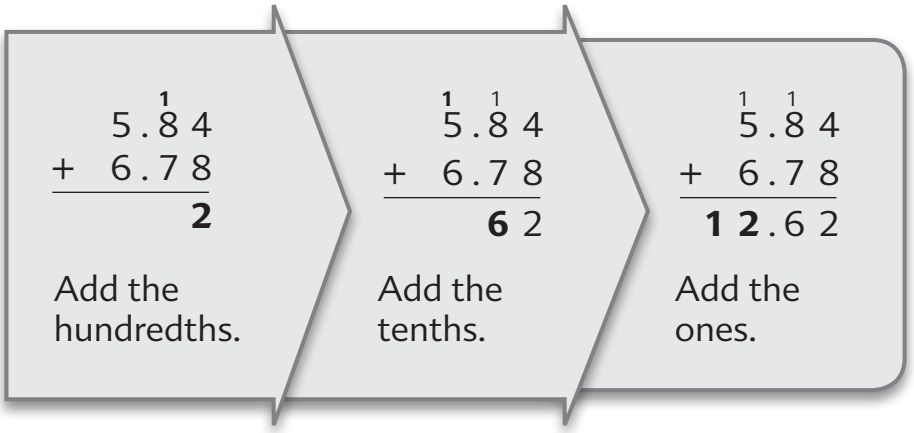
5. Round 103.821 to 2 decimal places.

103.821 ≈ 103.82

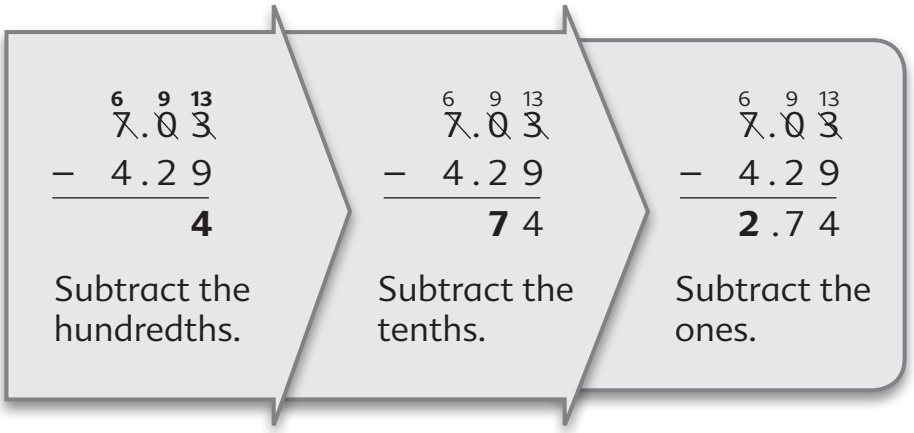


Add and Subtract Decimals

1. Add 5.84 and 6.78.



2. Subtract 4.29 from 7.03.



3. Estimate. Then find the value of 2.2 + 4.95.

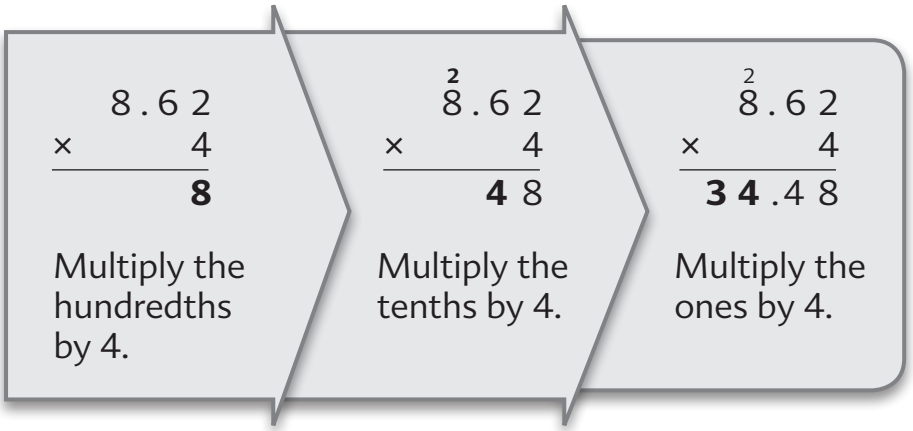
$2.2 + 4.95 \approx 2 + 5$
$= 7$
$2.2 + 4.95 = 7.15$

4. Estimate. Then find the value of 8.05 − 3.47.

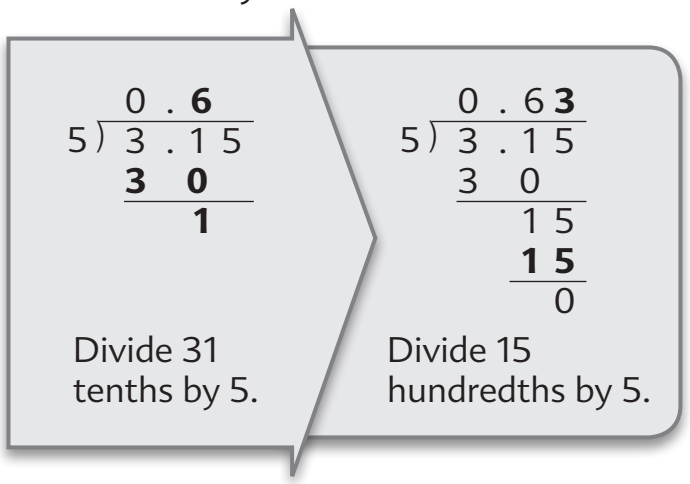
$8.05 - 3.47 \approx 8.10 - 3.50$
$= 4.60$
$8.05 - 3.47 = 4.58$

Multiply and Divide Decimals by a 1-Digit Whole Number

1. Multiply 8.62 by 4.



2. Divide 3.15 by 5.



3. Estimate. Then find the value of 3.12 × 4.

$3.12 \times 4 \approx 3 \times 4$

$= 12$

$3.12 \times 4 = 12.48$

4. Estimate. Then find the value of 14.6 ÷ 8.

$14.6 \div 8 \approx 16 \div 8$

$= 2$

$14.6 \div 8 = 1.825$

Multiplication by Tens, Hundreds, or Thousands

The value of a decimal is increased 10 times when multiplied by 10.

1. Multiply 0.425 by 10.

$0.425 \times 10 = 4.25$

$0.425 \times 10^1 = 4.25$

2. Multiply 0.425 by 20.

$0.425 \times 20 = 0.425 \times 2 \times 10$

$= 0.85 \times 10$

$= 8.5$

When a decimal is multiplied by 10, we move the decimal point 1 place to the right.

The value of a decimal is increased 100 times when multiplied by 100.

3. Multiply 3.806 by 100.

$3.806 \times 100 = 380.6$

$3.806 \times 10^2 = 380.6$

When a decimal is multiplied by 100, we move the decimal point 2 places to the right.

4. Multiply 3.806 by 500.

$3.806 \times 500 = 3.806 \times 5 \times 100$

$= 19.03 \times 100$

$= 1{,}903$

The value of a decimal is increased 1,000 times when multiplied by 1,000.

5. Multiply 4.782 by 1,000.

$4.782 \times 1{,}000 = 4{,}782$

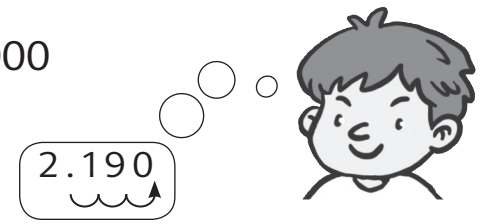
$4.782 \times 10^3 = 4{,}782$

6. Multiply 0.365 by 6,000.

$0.365 \times 6{,}000 = 0.365 \times 6 \times 1{,}000$

$= 2.19 \times 1{,}000$

$= 2{,}190$



Division by Tens, Hundreds, or Thousands

The value of a decimal is reduced 10 times when divided by 10.

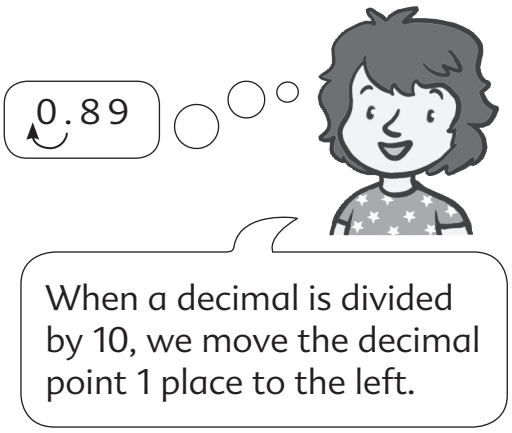
1. Divide 0.89 by 10.

$0.89 \div 10 = \frac{0.89}{10}$

$= 0.089$

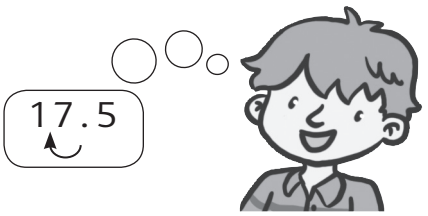
$0.89 \div 10^1 = 0.89 \times \frac{1}{10^1}$

$= 0.089$



2. Divide 52.5 by 30.

$$52.5 \div 30 = 52.5 \div 3 \div 10$$
$$= 17.5 \div 10$$
$$= 1.75$$

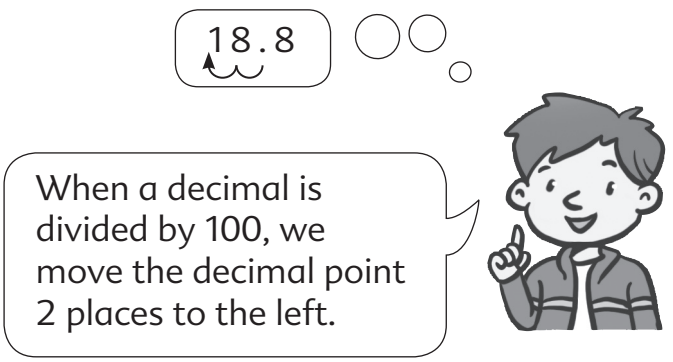


The value of a decimal is reduced 100 times when divided by 100.

3. Divide 18.8 by 100.

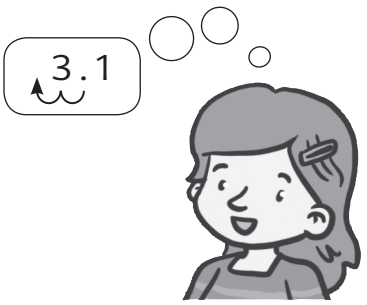
$$18.8 \div 100 = \frac{18.8}{100}$$
$$= 0.188$$

$$18.8 \div 10^2 = \frac{18.8}{10^2}$$
$$= 0.188$$



4. Divide 27.9 by 900.

$$27.9 \div 900 = 27.9 \div 9 \div 100$$
$$= 3.1 \div 100$$
$$= 0.031$$

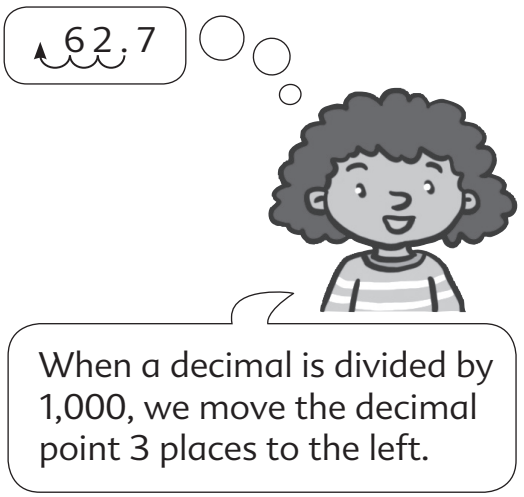


The value of a decimal is reduced 1,000 times when divided by 1,000.

5. Divide 62.7 by 1,000.

$$62.7 \div 1{,}000 = \frac{62.7}{1{,}000}$$
$$= 0.0627$$

$$62.7 \div 10^3 = \frac{62.7}{10^3}$$
$$= 0.0627$$



6. Divide 49 by 7,000.

$$49 \div 7{,}000 = 49 \div 7 \div 1{,}000$$
$$= 7 \div 1{,}000$$
$$= 0.007$$

Unit 8 : More Calculations

Friendly Notes

Multiplication by a 2-Digit Whole Number

Multiply 6.80 by 15.

$$\begin{aligned} 6.80 \times 15 &= 6.80 \times 10 + 6.80 \times 5 \\ &= 68.0 + 34.0 \\ &= 102 \end{aligned}$$

$6.80 \approx 7$

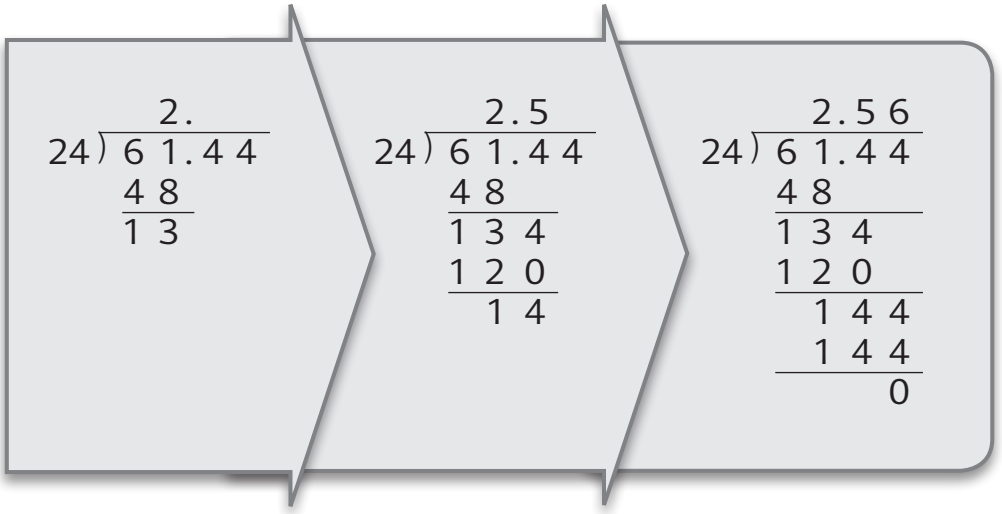
$$\begin{aligned} 6.80 \times 15 &\approx 7 \times 15 \\ &= 105 \end{aligned}$$

The estimate 105 is close to the answer 102. Therefore, the estimate is reasonable .

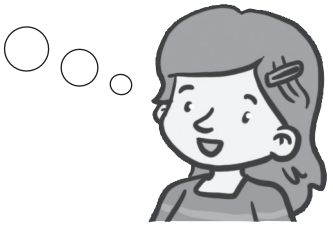
The estimate is reasonable as it is close to the answer.

Division by a 2-Digit Whole Number

Divide 61.44 by 24.



Estimate:
$61.44 \div 24 \approx 60 \div 20$
$= 3$



Multiplication by a Decimal

When a decimal is multiplied by 0.1, we move the decimal point 1 place to the left.

1. Multiply 25.6 by 0.1.

   $25.6 \times 0.1 = 2.56$

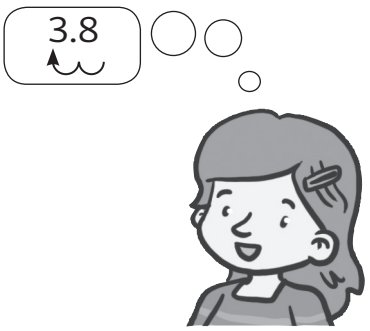
2. Multiply 34.2 by 0.5.

$34.2 \times 0.5 = 34.2 \times 5 \times 0.1$
$= 171 \times 0.1$
$= 17.1$

When a decimal is multiplied by 0.01, we move the decimal point 2 places to the left.

3. Multiply 3.8 by 0.01.

$3.8 \times 0.01 = 0.038$



4. Multiply 42.8 by 0.05.

$42.8 \times 0.05 = 42.8 \times 5 \times 0.01$
$= 214 \times 0.01$
$= 2.14$

5. Estimate. Then find the value of 8.2×0.09.

$8.2 \times 0.09 \approx 8 \times 0.09$
$= 0.72$

$8.2 \times 0.09 = 8.2 \times 9 \times 0.01$
$= 73.8 \times 0.01$
$= 0.738$

Division by a Decimal

1. Divide 49.28 by 0.7.

$49.28 \div 0.7 = 492.8 \div 7$
$= 70.4$

49.28 ÷ 0.7

$$\begin{array}{r} 70.4 \\ 7\overline{)492.8} \\ 49 \\ \hline 028 \\ 28 \\ \hline 0 \end{array}$$

2. Find the value of 8.648 ÷ 0.88 correct to 2 decimal places.

$8.648 \div 0.88 = 864.8 \div 88$
$= 9.827$
≈ 9.83

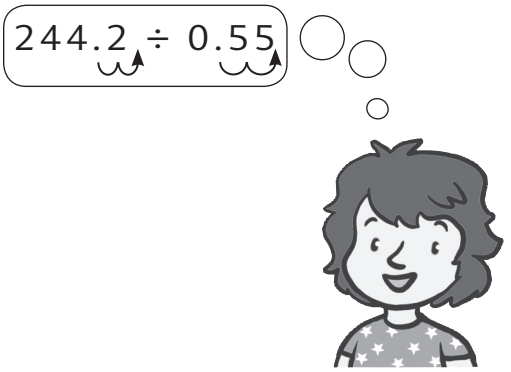
8.648 ÷ 0.88

$$\begin{array}{r} 9.827 \\ 88\overline{)864.800} \\ 792 \\ \hline 728 \\ 704 \\ \hline 240 \\ 176 \\ \hline 640 \\ 616 \\ \hline 24 \end{array}$$

3. Estimate. Then find the value of 244.2 ÷ 0.55.

$244.2 \div 0.55 \approx 250 \div 0.5$
$= 500$

$244.2 \div 0.55 = 24{,}420 \div 55$
$= 444$



Conversion of Measures

1 m = 100 cm	1 yd = 3 ft
0.1 m = 10 cm	1 ft = 12 in.
0.01 m = 1 cm	
1 km = 1,000 m	

1. Express 3.75 km in meters.

$3.75 \text{ km} = 3.75 \times 1{,}000$
$= 3{,}750 \text{ m}$

2. Express 42 in. in ft.

$42 \text{ in.} = \frac{42}{12} \text{ ft}$

$= 3\frac{1}{2} \text{ ft}$

1 kg = 1,000 g 1 lb = 16 oz
1 g = 0.001 kg

3. Express 320 g in kg.

$320 \text{ g} = \frac{320}{1{,}000} \text{ kg}$

$= \frac{8}{25} \text{ kg}$

4. Express 64 oz in lb.

$64 \text{ oz} = \frac{64}{16} \text{ lb}$

$= 4 \text{ lb}$

1 L = 1,000 ml
1 gal = 4 qt
1 qt = 2 pt
1 qt = 4 c

5. Express 0.58 L in ml.

$0.58 \text{ L} = 0.58 \times 1{,}000 \text{ ml}$

$= 580 \text{ ml}$

6. Express 10 qt in gal.

$10 \text{ qt} = \frac{10}{4} \text{ gal}$

$= 2.5 \text{ gal}$

7. Express 84 pt in qt.

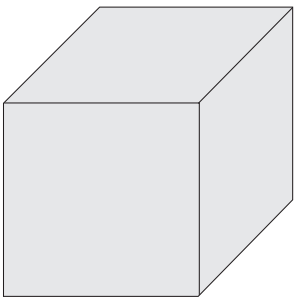
$84 \text{ pt} = \frac{84}{2} \text{ qt}$

$= 42 \text{ qt}$

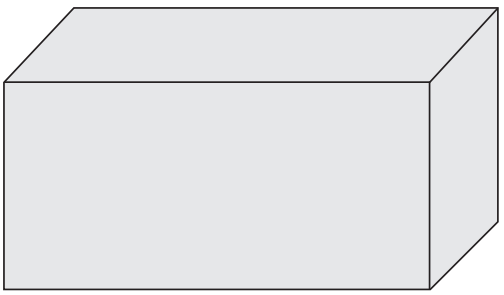
Unit 9 : Volume

Friendly Notes

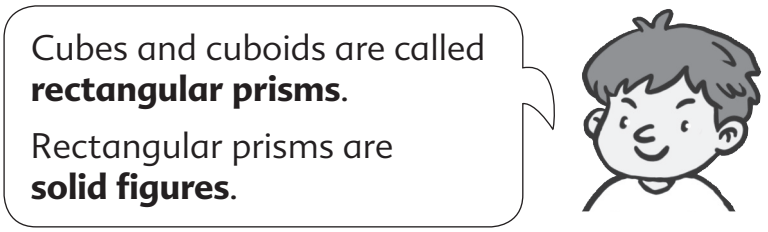
Cubic Units



All the edges of a **cube** are of equal length.
The sides are all squares.



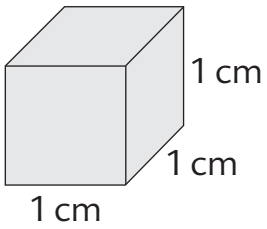
A **cuboid** has rectangular and/or square sides.



Volume of Rectangular Prisms

The **volume** of a solid is the amount of space it occupies.
The volume of a unit cube is 1 cubic unit.

Each edge of the cube is 1 cm long.
The volume of the cube is 1 cubic centimeter (cm^3).



The cubic centimeter (cm^3) is a unit of volume.
The cubic inch ($in.^3$), cubic foot (ft^3), and cubic meter (m^3) are other units of volume.

Volume of rectangular prism = length × width × height

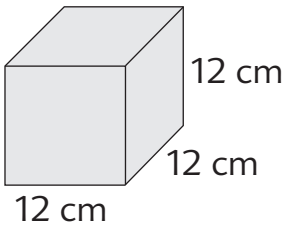
1. Find the volume of the rectangular prism which measures 18 cm by 20 cm by 10 cm.

   Volume of rectangular prism = 18 × 20 × 10
   = 3,600 cm^3

2. Find the height of the rectangular prism which has a base area of 24 m^2 and a volume of 120 m^3.

   $$\text{Height of rectangular prism} = \frac{\text{volume}}{\text{base area}}$$
   $$= \frac{120}{24}$$
   $$= 5 \text{ m}$$

3. Find the volume of the cube.



Volume of cube $= 12 \times 12 \times 12$
$= 1{,}728 \text{ cm}^3$

4. A rectangular container which measures 24 cm by 18 cm by 10 cm is $\frac{3}{4}$ filled with water. Find the volume of water in the container.

Volume of water in container $= \frac{3}{4} \times (24 \times 18 \times 10)$
$= 3{,}240 \text{ cm}^3$

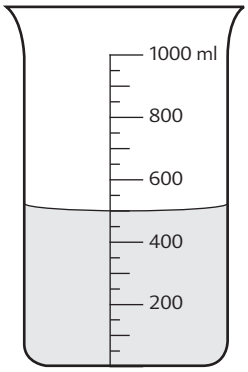
5. A rectangular tank 16 cm long and 15 cm wide is filled with water to a depth of 8 cm. If 240 cm^3 more water is needed to fill the tank, find the height of the tank.

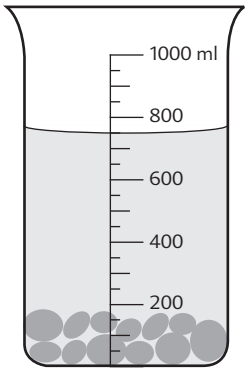
Volume of water in tank $= 16 \times 15 \times 8$
$= 1{,}920 \text{ cm}^3$

Volume of tank $= 1{,}920 + 240$
$= 2{,}160 \text{ cm}^3$

Height of tank $= \frac{2{,}160}{16 \times 15}$
$= 9 \text{ cm}$

Finding the Volume of a Solid





When a solid is placed into a container filled with water, the water level in the container will increase. We say that the volume of the water displaced by the solid is equal to the volume of the solid.

Volume of water in the beaker = 500 cm^3

Volume of water and the solid = 750 cm^3

Volume of the solid = 750 cm^3 – 500 cm^3

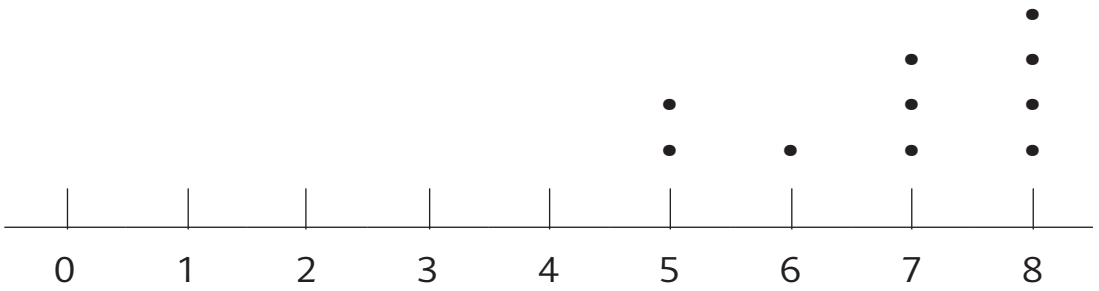
= 250 cm^3

Line Plots

The results of surveys can be organized in different ways to help us analyze the data more easily. We can use a line plot to present the data.

1. Seth conducted a survey to find out how many glasses of water students drink in a day. He then created a line plot to show his data clearly.

Number of glasses of water students drink in a day				
8	7	5	8	8
6	7	8	7	5



Number of glasses of water students drink in a day

(a) Look at the line plot and without calculating, estimate the average number of glasses of water each student drinks in a day.

(b) What is the average number of glasses the students drink in a day?

(a) The average estimated number of glasses of water each student drinks in a day is 7 glasses or 8 glasses.

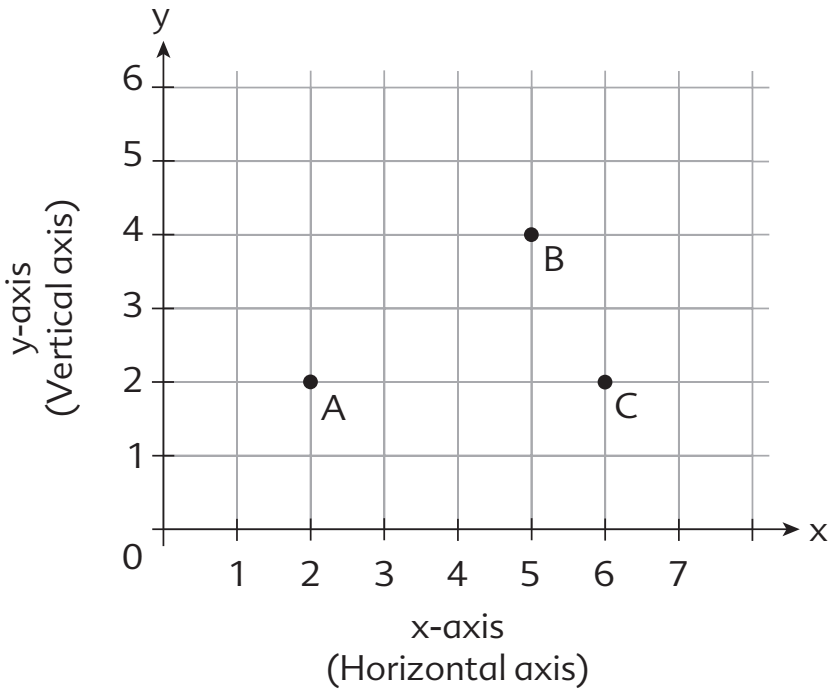
(b) $\text{Total} = (2 \times 5) + (1 \times 6) + (3 \times 7) + (4 \times 8)$
$= 10 + 6 + 21 + 32$
$= 69$

$\text{Average number of glasses of water} = 69 \div 10$
$= 6\frac{9}{10}$

Each student drinks an average of $6\frac{9}{10}$ glasses of water a day.

Coordinate Graphs

A coordinate grid has two axes. They are the **x-axis** (horizontal axis) and the y-axis (vertical axis).
The axes meet at the **origin** or the point (0, 0).



(2, 2) ⟶ two units from O along the x- and y- axes.

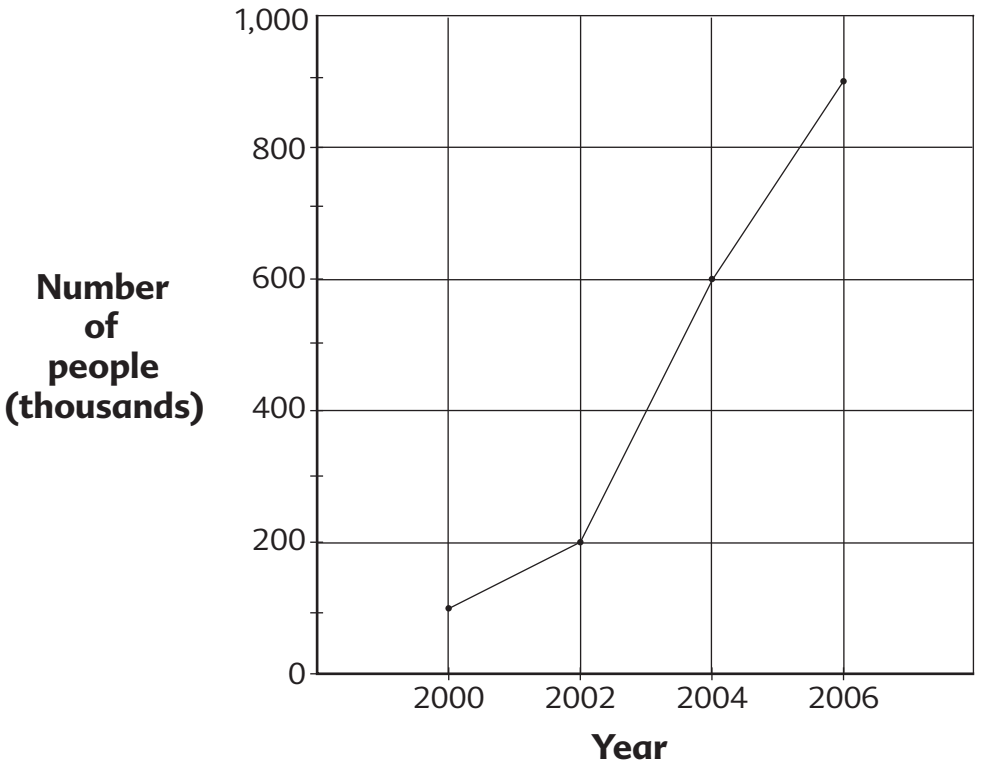
(5, 4) ⟶ five units from O along the x-axis, 4 units from O along the y-axis.

(2, 2), (5, 4), and (6, 2) are **ordered pairs**.
The numbers in an ordered pair are called the **coordinates**.
The first number is called the **x-coordinate** and the second number is called the **y-coordinate**.
Coordinates of A are (2, 2).
Coordinates of B are (5, 4).
Coordinates of C are (6, 2).

Line Graphs

Line graphs are used to represent data which changes over time.

1. The line graph shows the estimated number of people who own a cell phone in a city in the years from 2000 to 2006.



(a) When was the increase in the number of people who own the cell phone the greatest?

(b) Find the increase in the number of people who own a cell phone from 2000 to 2006.

(a) The greatest increase in the number of people who own a cell phone is between 2002 and 2004.

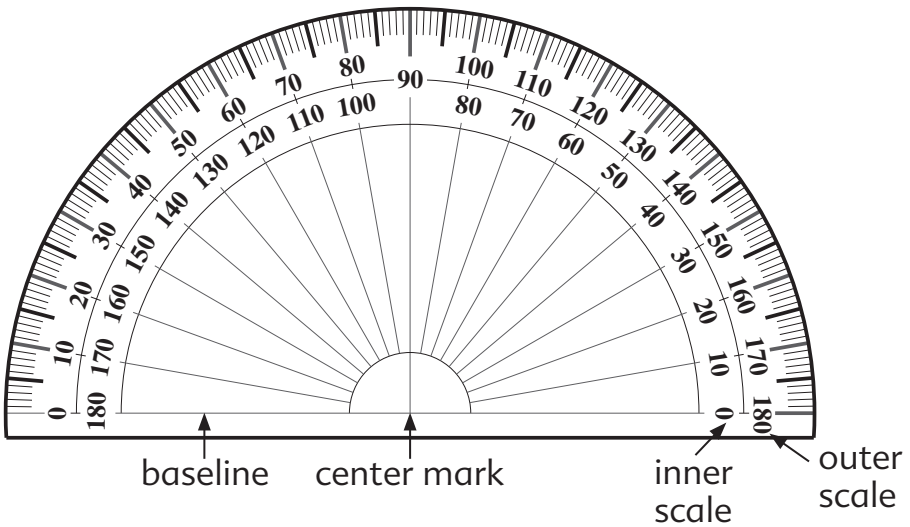
(b) 900,000 − 100,000 = 800,000
The increase in the number of people who own a cell phone from 2000 to 2006 is 800,000.

Unit 11 : Angles, Triangles, and Quadrilaterals

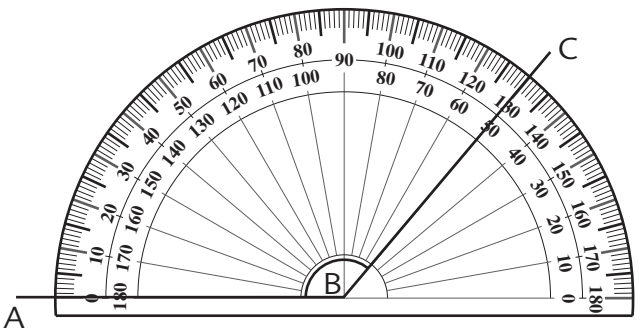
Friendly Notes

Looking Back

A **protractor** is used to measure angles.



1. Measure angle ABC.

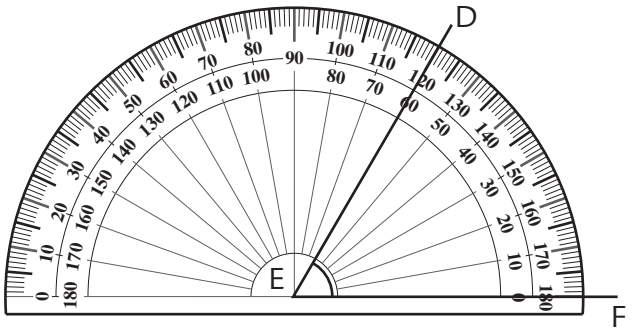


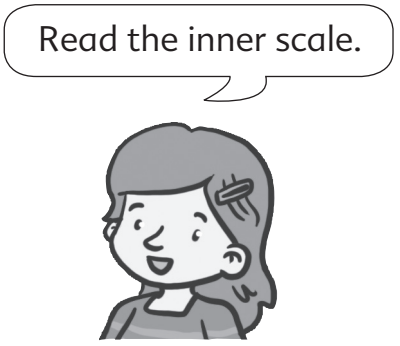
Place the baseline of the protractor on the horizontal line of the angle and make sure the center mark touches point B.

$m\angle ABC = 130°$

Read the outer scale.

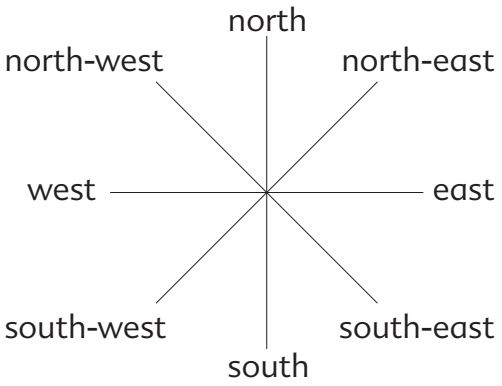
2. Measure ∠DEF.





m∠DEF = 60°

3.



(a) Sally starts facing north and turns clockwise 90°, which direction is she facing?

(b) Sally then turns counter clockwise to south-west. What angle does she turn through?

(c) After turning clockwise through 135°, Sally ends up facing south. Which direction was she facing at the start?

(a) She is facing east.

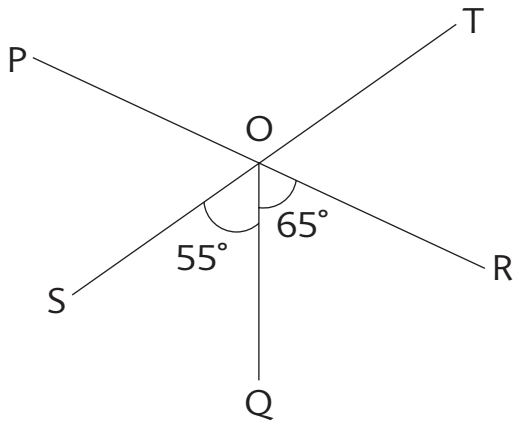
(b) She turns through 225°.

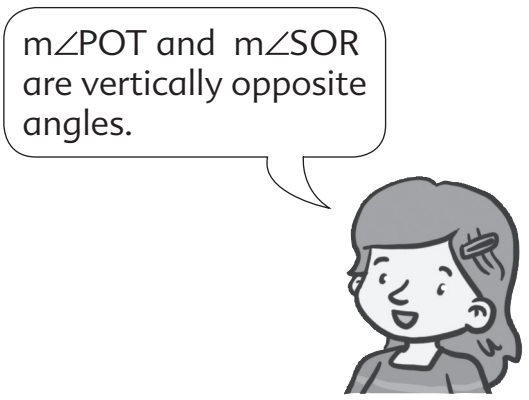
(c) She was facing north-east.

Finding Unknown Angles

Vertically opposite angles are equal.
The sum of the angles on a straight line is 180°.

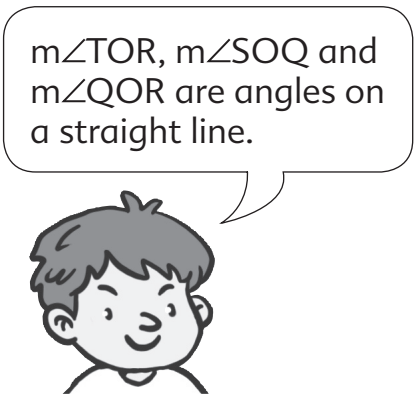
1. In the figure, POR and SOT are straight lines. Find
   (a) m∠POT, and
   (b) m∠TOR.



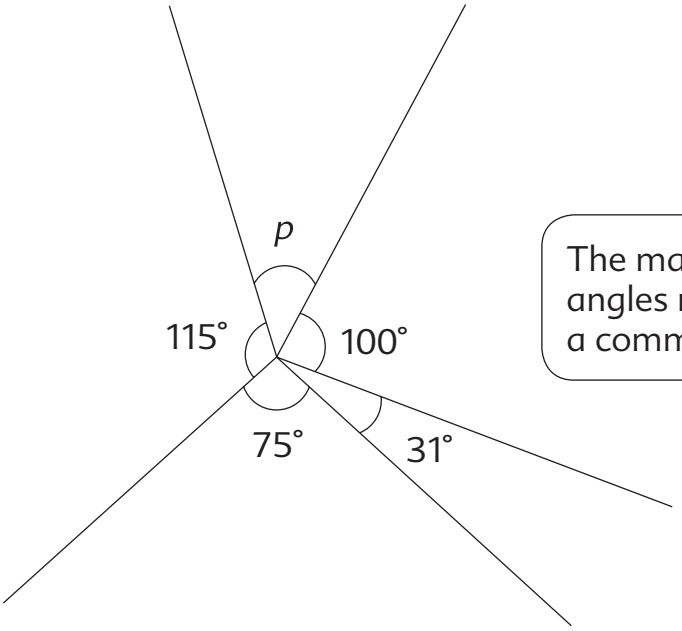


(a) $m\angle POT = m\angle SOR$
$= 55° + 65°$
$= 120°$

(b) $m\angle TOR = 180° - 55° - 65°$
$= 60°$



2. Find m∠p in the figure.

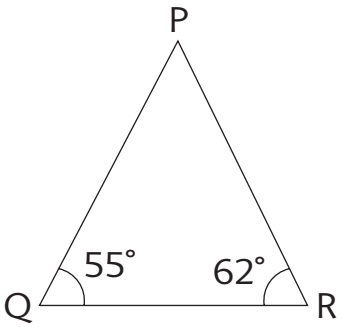


$$\begin{aligned} m\angle p &= 360^\circ - 100^\circ - 31^\circ - 75^\circ - 115^\circ \\ &= 39^\circ \end{aligned}$$

Finding Unknown Angles in Triangles

The three angles of a triangle add up to 180°.

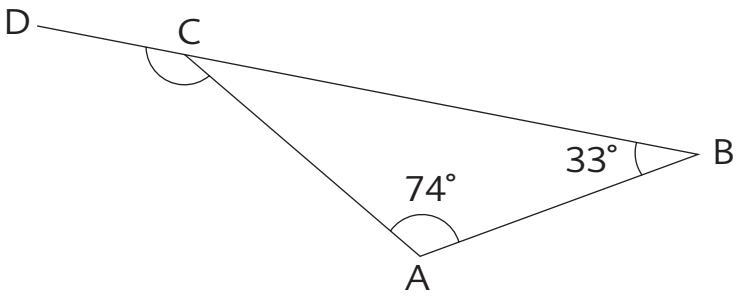
1. In triangle PQR, m∠PQR = 55° and m∠PRQ = 62°. Find m∠QPR.



$$m\angle QPR = 180° - 55° - 62°$$
$$= 63°$$

The exterior angle of a triangle is equal to the sum of its interior opposite angles.

2. In triangle ABC, BC is extended to D, m∠CAB = 74°, and m∠ABC = 33°. Find m∠ACD.

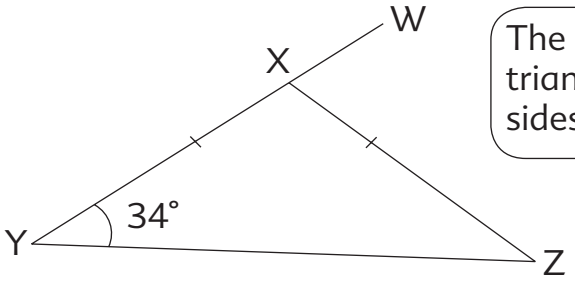


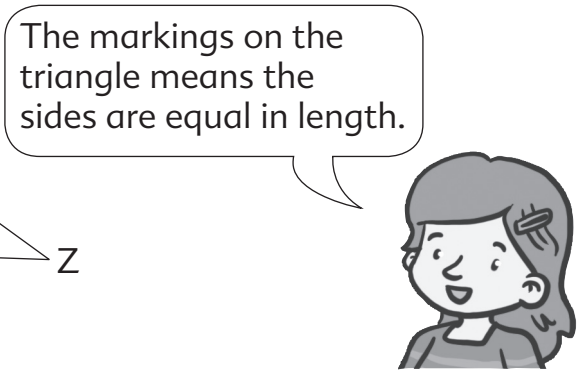
$$m\angle ACD = 74° + 33°$$
$$= 107°$$

Isosceles and Equilateral Triangles

An **isosceles** triangle has 2 equal sides. The angles opposite the equal sides are equal.

1. In triangle XYZ, XY = XZ, m∠XYZ = 34°, and WXY is a straight line. Find m∠WXZ.



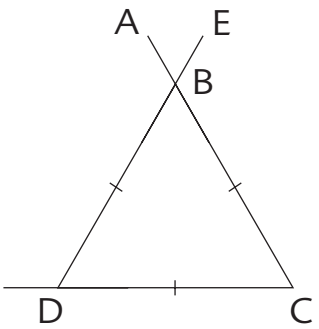


$m\angle XZY = m\angle XYZ$
$= 34°$

$m\angle WXZ = 34° + 34°$
$= 68°$

An **equilateral** triangle has 3 equal sides and 3 equal angles. Each angle is 60°.

2. In the figure, EBD and ABC are straight lines. Find m∠ABE.



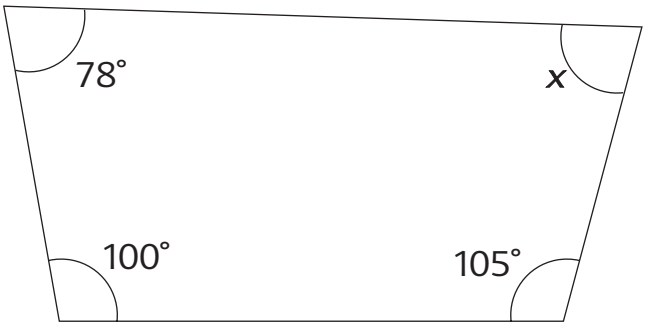
$m\angle DBC = 60°$

$m\angle ABE = m\angle DBC$
$= 60°$

Finding Unknown Angles in Quadrilaterals

The angles of a quadrilateral add up to 360°.

Find m∠x in the quadrilateral.

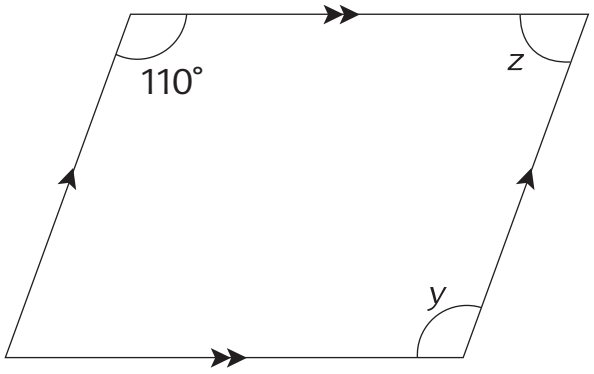


$$m\angle x = 360° - 78° - 100° - 105°$$
$$= 77°$$

Parallelograms, Rhombuses, and Trapezoids

The opposite angles of a parallelogram are equal.
Each pair of angles between two parallel sides add up to 180°.

Find m∠y in the parallelogram.



m∠y = 110°

m∠z = 180° − 110°
= 70°

Unit 12 : Percentage

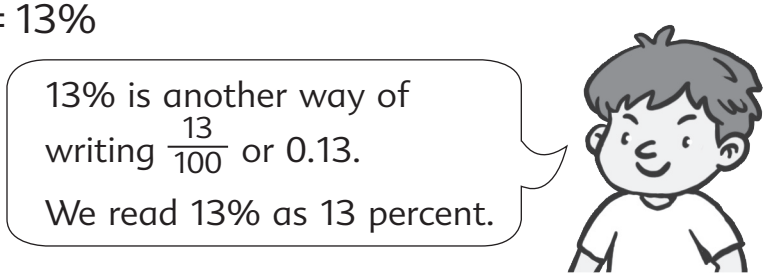
Friendly Notes

Percent

A **percentage** is a way of expressing a fraction with a denominator of 100.
A percentage is indicated by the symbol, % (percent).

1. Write 13 out of 100 as a percentage.

   13 out of 100 = 13%



2. Express 15% as a fraction in its simplest form.

$$15\% = \frac{15}{100}$$
$$= \frac{3}{20}$$

3. Express 88% as a decimal.

$$88\% = \frac{88}{100}$$
$$= 0.88$$

4. Express 0.45 as a percentage.

$$0.45 = \frac{45}{100}$$
$$= 45\%$$

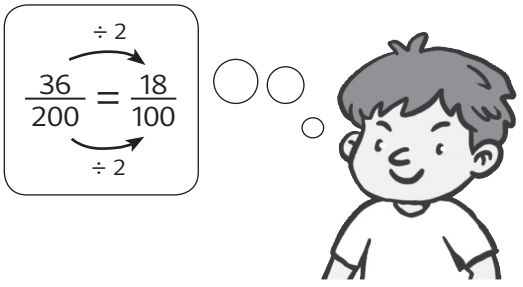
Writing Fractions as Percentages

Express $\frac{36}{200}$ as a percentage.

Method 1:

$$\frac{36}{200} = \frac{18}{100}$$

$$= 18\%$$



Method 2:

$$\frac{36}{200} = \frac{36}{200} \times 100\%$$

$$= 18\%$$

Percentage of a Quantity

There are 45 children in a class. 40% of the children are girls. How many boys are there in the class?

$100\% - 40\% = 60\%$

60% of the children are boys.

$$60\% \text{ of } 45 = \frac{60}{100} \times 45$$

$$= 27$$

There are 27 boys in the class.